Final Exam Introduction to Logic 2018/19 (AI and MA + Guests)

Wednesday 30 January, 2019, 9 – 12 AM

Instructions: Read Carefully

- ☞ Put the version of the course that you are registered in at the top of the first page (either "AI" or "Math + Guests").
- ☞ Only write your student number at the top of the exam, not your name, so that we can grade anonymously. Also put your student number at the top of any additional pages.
- ☞ Leave the first ten lines of the first page blank (for the calculation of your grade).
- ☞ Use a blue or black pen (so no pencils, no markers, no red pens).
- ☞ With the regular exercises, you can earn 90 points. By writing your student number and tutorial group on all pages, you earn a first 'free' 10 points. With the bonus exercise, you can earn an additional 10 points. The total exam grade is: *(the number of points you earned with the regular and bonus exercises + the first 'free' 10) divided by 10, with a maximum grade of 10.*
- ☞ The final grade F for the course is computed as

$$F = 0.08 \cdot H_1 + 0.16 \cdot H_2 + 0.16 \cdot M + 0.60 \cdot E.$$

  Here H_1 is the grade for homework assignment 1, H_2 is the grade for homework assignment 2, M is the midterm grade, and E is the grade for this final exam.

Good Luck!

1: Translating to propositional logic (10 points) Translate the following sentences to *propositional logic*. Atomic sentences are represented by uppercase letters. Provide one translation key for both sentences.

a. Libraries are nice study places if they are quiet places, and they are nice study places only if they are quiet places.

b. Either libraries are both nice study places and cosy meeting places, or they are quiet places.

2: Translating to first-order logic (10 points) Translate the following sentences to *first-order logic*. Provide one translation key for both sentences. The domain of discourse is the set of all cities.

a. All cities are linked to Rome and Rome is linked to all cities.

b. If there is a city that is more beautiful than Rome, then that city is either Venice or Florence, and no other city than that city is more beautiful than Rome.

3: Formal proofs (20 points) Give formal proofs of the following inferences. In items c and d, P, Q and R are predicate symbols. Do not forget the justifications. Only use the Introduction and Elimination rules and the Reiteration rule.

a.
$$\begin{array}{|l} (A \wedge B) \vee (C \wedge D) \\ \hline (A \vee C) \wedge (B \vee D) \end{array}$$

b.
$$\begin{array}{|l} A \leftrightarrow (\neg B \vee C) \\ \neg A \\ \hline B \wedge \neg C \end{array}$$

c.
$$\begin{array}{|l} \forall x \forall y (Q(x) \to R(y)) \\ \hline \exists x Q(x) \to \forall y R(y) \end{array}$$

d.
$$\begin{array}{|l} \exists y \exists x (x = y \wedge P(x, y)) \\ \hline \exists x P(x, x) \end{array}$$

4: Truth tables (10 points) Answer the following questions using truth tables. Write down the complete truth tables and motivate your answers. Order the rows in the truth tables as follows:

A	B	C	...
T	T	T	...
T	T	F	...
T	F	T	...
T	F	F	...
F	T	T	...
F	T	F	...
F	F	T	...
F	F	F	...

Small(a)	Medium(b)	Smaller(a, b)	...
T	T	T	...
T	T	F	...
T	F	T	...
T	F	F	...
F	T	T	...
F	T	F	...
F	F	T	...
F	F	F	...

a. Check with a truth table whether the following formula is a tautology.

$$(A \vee (B \to C)) \to (B \vee (A \to C))$$

b. Check with a truth table whether the following formula is Tarski's World-(TW-)possible. Indicate clearly which are the spurious row(s), if there are any.

$$(\mathsf{Small(a)} \leftrightarrow \mathsf{Medium(b)}) \wedge \neg(\mathsf{Smaller(a,b)} \vee \neg\mathsf{Medium(b)})$$

5: Normal forms propositional logic (5 points) Provide a conjunctive normal form (CNF) of the following formula. Show all of the intermediate steps.

$$(\neg C \wedge B) \leftrightarrow \neg(A \vee \neg B)$$

6: Normal forms for first-order logic and Horn sentences (10 points)

a. Provide a Prenex normal form of the following sentence. Show all intermediate steps.

$$\exists x \forall y R(y, x) \vee \neg \forall z \exists x \exists y (Q(z, y, x) \wedge \neg P(x))$$

b. Provide a Skolem normal form of the following sentence. Show all intermediate steps.

$$\forall x \forall y \exists z \forall w \exists v ((R(x, z) \to Q(w, v, z)) \vee (\neg R(y, z) \wedge P(w)))$$

c. Check the satisfiability of the following Horn sentence.

$$A \wedge \neg B \wedge (\neg A \vee B \vee \neg C) \wedge (C \vee \neg E) \wedge (\neg A \vee \neg D \vee E)$$

Use the Horn algorithm and indicate the order in which you assign truth values to the atomic sentences. If you prefer the conditional form, you may also rewrite the formula with $\to$ and then use the satisfiability algorithm for conditional Horn sentences.

7: Sets and relations (8 points) Given are the following five sets: $A = \{2\}$, $B = \{1,2\}$, $C = \{\langle 1,2\rangle\}$, $D = \{2, \langle 2,2\rangle\}$, and $R = \{\langle 2,2\rangle, \langle 1,2\rangle\}$. For each of the following statements, determine whether it is true or false. You are not required to explain the answer.

a. $\emptyset \in A$

b. $A \subseteq C$

c. $A \subset D$

d. $(B \setminus C) \cup A = B$

e. $A \subset (R \cap C) \cup B$

f. $(R \setminus C) \cup (A \setminus C) = D$

g. $\emptyset = D \cap C$

h. $B \setminus A \in C$

8: Translating function symbols (7 points) Translate the following sentences using the translation key provided. The domain of discourse is the set of all persons.

a: Russell	supervisor(x): x's supervisor
b: Wittgenstein	StudentOf(x, y): x is a student of y

a. Russell is the supervisor of Wittgenstein, but Wittgenstein is nobody's supervisor.

b. For each pair of two persons, the first person is the supervisor of the second one if and only if the second person is a student of the first person.

c. Every supervisor of a supervisor is a student of someone.

9: Semantics (10 points)

Let a model $\mathfrak{M}$ with domain $\mathfrak{M}(\forall) = \{1, 2\}$ be given such that

- $\mathfrak{M}(a) = 1$
- $\mathfrak{M}(P) = \{1\}$
- $\mathfrak{M}(R) = \{\langle 1,2\rangle, \langle 2,1\rangle, \langle 2,2\rangle\}$

Let h be an assignment such that:

- $h(x) = 1$
- $h(y) = 2$

Evaluate the following statements. Follow the truth definition step by step.

a. $\mathfrak{M} \models R(a, x) \vee (R(y, y) \wedge \neg P(a))[h]$

b. $\mathfrak{M} \models \exists x \forall y (R(x, y) \to P(y))[h]$

c. $\mathfrak{M} \models \forall x (P(x) \to \exists y(\neg P(y) \wedge R(y, y)))[h]$

10: Bonus question (10 points) Give a formal proof of the following inference. Don't forget to provide justifications. Only use the Introduction and Elimination rules and the Reiteration rule.

$$\begin{array}{|l} \neg\forall x \exists y \forall z \neg R(x, y, z) \\ \hline \exists u \forall v \exists w R(u, v, w) \end{array}$$